A LOCUS PROBLEM SOLVING IN DYNAMIC GEOMETRY ENVIRONMENT

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INTRODUCTION

In recent decades, computers have become one of the most important tools of technology-supported education. The most commonly used mathematical software programmes in mathematical education – especially in geometry teaching - are dynamic geometry programmes because of their straight connection to visualisation of geometrical relationships. Working in a dynamic geometry environment (DGE) can encourage students to investigate in geometry. In DGE, students can experiment by dragging the geometrical objects they create and thus deduce their properties and generalise them. The main purpose of this paper is to present some indications of how DGE (especially using GeoGebra software), can offer prospective teachers investigation opportunities and experimentation to solve locus problems in geometry, as well as support understanding and building the proofs.
In geometry, a set of all points satisfying one or more specified conditions is called a locus of points or simply a locus. A general description of the construction of a locus given by a geometric configuration is the following. A locus is consisting of different positions of tracing point T satisfying a given property. This property usually is given by relationship between a moving point M and point T, where M is a point on the one-dimensional object. While M moves along the one-dimensional object, T traces the locus. Thus the locus is defined as the image of an object under an application or transformation: the function that transforms the “mover” into the “tracer”. This description also corresponds with how the dynamic geometric software is able to build the image points, i.e. the locus.
Problem-solving is a process of engaging in a task or a situation for which there is no obvious or immediate solution. We divided the problem-solving process on constructive geometry tasks into the following phases:

1. Visualisation of the problem (sketch),
2. Investigation, exploration phase,
3. Conjecture, hypothesis creation,
4. Verification (optionally generalisation),
5. Explanation and proof.

We posed the research question: How can GeoGebra efficiently help in the solution process of a demanding geometric locus problem?
METHODOLOGY

• Our research method was to follow and observe students’ work on the plane geometry problem with the usage of qualitative research design. Prospective mathematics teachers solved a demanding problem on locus of points in pen and paper environment with the possibility of working also in a dynamic geometry environment (DGE).

• The participants of the case studies were selected according to their interest in Euclidian geometry, the capability of using GeoGebra, and eloquence to facilitate the data collection process. We collected the data from four sources: paper worksheets, GeoGebra files, audio records, and a questionnaire. The audio records (we asked students to “think loudly”) during the solution process allow us to identify common key points in paper worksheets and GeoGebra files.

• Using the navigation bar in the students’ GeoGebra files we are able to reconstruct their solution step by step and analyse them.
• We conducted four individual sessions with four participants from two universities preparing mathematics teachers in Slovakia. The first subject of their study was Mathematics and the second one was one of Hungarian Language and Literature, Art, and Computer Science. The individual sessions took place in a school classroom each time with only one participant and the researcher presence. The problem they were required to solve is described in the next chapter. During the approximately 30-40 minute work on the task, the participant could ask for help, which usually consisted of a leading question or given some visual attention to a part of his/her prepared GeoGebra picture. After completing the task, the students had to answer a questionnaire.
ORGANISING THE CASE STUDIES

• Our questionnaire consisted of the following questions:

1. In which phase of problem solving was GeoGebra the most helpful?
   A) Data sketching and a better understanding of the task.
   B) To see the relationships between the given data.
   C) To formulate generalisations (the possibility to observe multiple cases with movement).
   D) To prove the discovered relationships.

2. For what type of geometric task did you find it most beneficial to use GeoGebra?
   A) Discovering relationships between shapes or parts of shapes.
   B) To determine the locus of points.
   C) Proofs.
The students solved the following task: What is the locus of the orthocentres of such triangles ABC, where side AB has constant length $c$ and also the size of angle ACB = $\gamma$ is constant?

BRIEF SOLUTION: Vertex C of triangle ABC is always on the circle which forms the locus of equiangular points to segment AB. The perpendiculars to sides AC and BC form the same angle $\gamma$ as it was given for angle ACB, so the locus of orthocentres is the shifted arc of the locus of equiangular points by vector CD.
OBSERVATIONS AND RESULTS

1. SOLUTION OF STUDENT A

- Student A got an idea to study what kind of circle could be the solution after seeing the relevant trace of orthocentres in GeoGebra.

The first sketch and the relevant picture in GeoGebra
OBSERVATIONS AND RESULTS

2. SOLUTION OF STUDENT B

• Student B composed a pen sketch, he guessed that the locus is a part of a circle but he was not able to explain the background of this idea.

• As a second attempt, he constructed a circle through A and B and then he chose the point C on the given circle. Changing the position of point C he could follow the trace of the orthocentre.

Hint: See the angles in quadrangle BMEC
OBSERVATIONS AND RESULTS
3. SOLUTION OF STUDENT C

• Student C began her solution with GeoGebra sketch.

• Her first attempt was not useful but the second one was smart. According to the smart sketch using the trace function, she could immediately realise that the locus she is looking for is a part of a circle.

• Student C invented the essence of the proof, she needed only complete the details with the help of the researcher.
Student D began her investigation on locus of points by making a pen sketch.

She correctly described the locus of vertices C of triangle ABC but she supposed that the locus in question would depend on the inscribed angle $2\gamma = \omega$. 
OBSERVATIONS AND RESULTS

4. SOLUTION OF STUDENT D

• Investigation in GeoGebra (especially for obtuse triangle) persuaded her that that was a mistake.

• Finally, she was the only one who found the locus correctly.
ANALYSIS OF PROBLEM-SOLVING

Atomic analysis in Hejný’s sense is a very deep and detailed analysis of mistakes and solution steps, so we try to show only some fragment of it here. The atomic analysis points out what ideas, either wrong or right, led the student’s thought to the final solution of the problem. Here we put the steps briefly into the next table to make them easy to overview.

<table>
<thead>
<tr>
<th></th>
<th>Step 1</th>
<th>Step 2</th>
<th>Step 3</th>
<th>Step 4</th>
<th>Step 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student A</td>
<td>Pen sketch of one case</td>
<td>Attempt to follow more cases</td>
<td>Plenty of guesses, see SA2</td>
<td>GeoGebra sketch, tracing</td>
<td>Proof with the help</td>
</tr>
<tr>
<td>Student B</td>
<td>Partly good guess (Figure 3)</td>
<td>Difficulty with GeoGebra sketch</td>
<td>Trace of the orthocentre</td>
<td>Investigation</td>
<td>Proof with the help</td>
</tr>
<tr>
<td>Student C</td>
<td>Inapplicable GeoGebra sketch</td>
<td>GeoGebra sketch, tracing</td>
<td>Estimation of the locus</td>
<td>Investigation</td>
<td>Almost full proof</td>
</tr>
<tr>
<td>Student D</td>
<td>The most precise sketch</td>
<td>Stipulating the locus</td>
<td>Investigation (also in special cases)</td>
<td>Almost full proof</td>
<td>Finishing proof with the small help</td>
</tr>
</tbody>
</table>
QUESTIONNAIRE RESPONSES

• Findings from the questionnaire say that according to the students participated, GeoGebra was the most helpful when they needed to see relationships and to formulate generalisations.

• Student D affirmed that “due to possible movement, I could see cases that can occur and also the connections between the elements”.

• The participants picked out different types of tasks in which they found it more beneficial to use GeoGebra – three of them chose possibility A) Discovering relationships between shapes or parts of shapes. Three participants (as prospective mathematics teachers) emphasised the usefulness of GeoGebra not only in learning but also in teaching geometry.
CONCLUSIONS

• During the locus problem solving we observed that in DGE, all participating students acquired understanding through verifying their conjectures that the locus was a part of a circle, and in turn, this comprehension activated further curiosity to explain why this particular result was correct.

• Students A and B finished the task with the active help of the researcher.

• A complete solution is from student D, who found the appropriate part of the circle adequate to the condition of the task using GeoGebra.

• Students A, B and C unanimously stated that they would probably not be able to find the locus without GeoGebra. Our observations also support this sentence, see for example SA3 and SB3.

• According to responses to the questions posed in our questionnaire, we learned that the possibility of investigation in motion was the most appreciated property of GeoGebra during problem-solving.
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THANK YOU FOR YOUR ATTENTION